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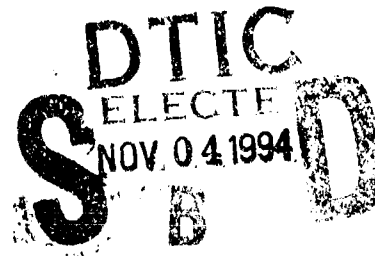
Comparison of Meteorological Data With Fitted Values Extracted From Projectile Trajectory

Gene R. Cooper
Kevin S. Fansler

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13. ABSTRACT   *(Maximum 200 words)*

In this report, the atmospheric conditions are found by knowing only the projectile's flight trajectory and its flight coefficients together with the initial atmospheric conditions on the ground. The test trajectories were generated as solutions of the modified point mass (MPM) equations of motion. The correct atmospheric conditions for the generated flight trajectory are obtained from data collected during a weather balloon flight. A nonlinear least squares method was then used to fit the MPM equations to the test trajectory by varying the meteorological parameters. Density, sound speed, and wind profiles agreed well. Further tests of the method involved the flight coefficients, which we perturbed to observe the corresponding variation for the fitted values. Also, Gaussian noise was introduced onto the trajectory values to simulate the uncertainty in trajectory measurements. The results of the analysis shows that the whole trajectory should be fitted, and not small trajectory segments, to obtain accurate atmospheric parameters with the location precision provided by current radar and Global Positioning Satellite System (GPS) techniques.

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TABLE OF CONTENTS

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LIST OF FIGURES

LIST OF TABLES

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1. INTRODUCTION

A better knowledge of atmospheric conditions could improve the accuracy of artillery. Currently, weather balloons are used to gather the temperature, pressure, and wind velocity as a function of height above the earth's surface. As the weather changes, the information becomes stale and contributes significantly to the artillery round's error budget (Matts and Ellis 1990). Ideally, the atmospheric conditions need to be known immediately before a round is fired.

Since the trajectory of a projectile depends upon the atmospheric conditions, among other things, the problem can be turned around to determine the atmospheric conditions by knowing the trajectory of a fired projectile. More specifically, a nonlinear least squares method can be used with the trajectory data, if the equations of motion are assumed with a knowledge of the projectile's flight coefficients, to yield a best fit to the density, temperature, and the wind velocity as a function of altitude. Thus, the atmospheric conditions would be determined soon after the first shot was fired, the elapsed time for the extraction of the meteorological results depending on the computer's speed and the efficiency of the solution algorithm. The atmospheric quantities could subsequently be used with the artillery piece swung to a new azimuth.

As a first step, Cooper and Bradley (1991) posed a problem that did not use atmospheric data from a balloon flight; to use complicated experimental data would pose a harder problem. Their "observed" trajectory was calculated from the modified point mass (MPM) equations of motion (Lieske and Reiter 1966, Bradley 1990) combined with the 1966 U. S. Standard Atmosphere (STAT) and linear wind profiles. The STAT assumes a linear temperature with altitude and a simple relationship between the temperature and density. To facilitate fitting the manufactured trajectory data, Bradley (1990) revised the MPM equations so that current fitting techniques could be adopted to iterate the air temperature gradient with respect to height and the wind velocity as fittable parameters. FINLIE (Bradley 1981), a nonlinear curve-fitting method, then fitted the revised MPM equations to the generated data by adjusting these parameter values. When the fitted values for trajectory, air density, air temperature, and wind profiles were compared with the generated corresponding values, the agreement was found to be excellent.

The program to implement the least-squares fit is quite complicated and uses a large amount of time on a computer's central processing unit (CPU). A FORTRAN subroutine was constructed for FINLIE that defines the original set of equations plus an auxiliary set. The auxiliary equations involve partial derivatives of the parameters to be fitted. Some of these equations are exceedingly complicated and the chances of error in obtaining these equations

1

are large if paper and pencil were to be used. To minimize errors, a software package called MACSYMA (MACSYMA 1983) was used. MACSYMA determined the partial differential expressions from the original equations and translated them to readily usable FORTRAN code.

In another approach that has not yet been implemented, the temperature and density from the most recent atmospheric data are assumed correct and the shell trajectory is then used to calculate the wind velocities (Pedersen 1994). Since only the wind velocity needs to be determined, the calculations should proceed rapidly. The possibility exists that the temperature and density could have been measured a few hours ago, thus introducing added uncertainties with time. Possible ways of modeling the density and temperature, given changes in the ground meteorological values, are being explored.

The work described in this report follow on and expand that of Cooper and Bradley (1991). Instead of using an idealized atmospheric model with the wind velocity being linear with height above the ground, the atmospheric data from a weather balloon were used as input to generate the test trajectory. The test trajectory was then fitted over segments to obtain approximations to the wind velocities, density, and temperature. Error studies were also performed. In practice, the flight coefficients are known only approximately. This uncertainty was studied by perturbing the accepted flight coefficient values to see how the meteorological values varied. Also, white noise was introduced onto the trajectory values to simulate the uncertainty in trajectory measured by radar or the global positioning satellite system (GPS) techniques.

2. TRAJECTORY EQUATIONS AND COORDINATE SYSTEM

The construction of firing tables depends significantly upon the use of the MPM trajectory model. More complex models are used only for special cases, and the complete data set to construct these more complex models is not commonly available. The 6-degree-of-freedom equations have time derivatives that do not appear in an explicit factored form. FINLIE, however, uses only equations with derivatives that are factored. Thus, the MPM equations will be used here in a different but almost equivalent form (Bradley 1990) than is commonly used in the Firing Tables Branch (Lieske and Reiter 1966).

Bradley's formulation for the MPM model can be re-written in the form

$$\vec{U} = (D - A)\vec{V} + \vec{G}_A$$

(1)

2

in which

$$D = \frac{h_a(\vec{G} \bullet \vec{V})}{(1 + h_a)V^2}$$

$$A = \left(\frac{\rho S \ell}{2m} C_D\right)\left(\frac{V}{\ell}\right)$$

$$\vec{G}_A = \frac{1}{1 + h_a}\left[\vec{G} + \frac{h_L(\vec{G} \times \vec{V})}{(1 - h_M)V}\right]$$

$$h_L = k_a^2\left(\frac{C_{L_\alpha}}{C_{M_\alpha}}\right)\left(\frac{\dot{\phi}\ell}{V}\right)$$

$$h_M = k_a^2\left(\frac{C_{\cdot \cdot \alpha}}{\cdot_{l\alpha}}\right)\left(\frac{\dot{\phi}\ell}{V}\right)^2$$

$$h_a = \frac{h_L^2}{1 - h_M} - h_M$$

and in which $\vec{U}$ = projectile velocity with respect to the earth

$\vec{V} = \vec{U} - \vec{W}$ = projectile velocity with respect to the air

$\vec{W}$ = wind velocity with respect to the earth

$\vec{G}$ = the sum of the gravity and Coriolis accelerations

$\dot{\phi}$ = axial spin, rad/s.

(All symbols are defined in the List of Symbols.) The axial spin $\dot{\phi}$ is obtained as the solution of a simplified roll equation:

$$\ddot{\phi} = -BC_{\ell_p}\left(\frac{\dot{\phi}\ell}{V}\right) \tag{2}$$

in which

$$B = k_a^{-2}\left(\frac{\rho S \ell}{2m}\right)\left(\frac{V}{\ell}\right)^2.$$

The aerodynamic coefficients C_{L_α}, C_{M_α}, $C_{N_{p\alpha}}$, and C_{ℓ_p} in Eqs.(1) and (2) are tabulated as functions of Mach number. The drag coefficient C_D depends on both Mach number and the yaw of repose, $\vec{\alpha}_e$. In particular, C_D is assumed to have the form

$$C_D = C_{Do} + C_{D2}|\vec{\alpha}_e|^2 \tag{3}$$

where C_{Do} is a function of Mach number and C_{D2} is a constant.

The yaw of repose can be computed from the relation

$$\vec{\alpha}_e = \frac{\dot{\phi}\,\vec{G}_A \times \vec{V}}{BV^2C_{M_\alpha}}. \tag{4}$$

3

A convenient coordinate system is needed for describing the motion of a projectile along its trajectory. Following Cooper and Bradley's (1991) treatment, we assume that the launch point is at sea level. Then we set our origin at the launch point. We then define a right-handed Cartesian system as follows: the 1- and 3-axes form a plane tangent to the earth at the origin; the 2-axis is perpendicular to this plane, positive upward, and the 1-axis is chosen so that the velocity $\vec{U}$ at time zero is in the 1-2 plane. Then, the projectile's position vector $\vec{X}$ with respect to the earth can be written in component form as

$$\vec{X} = (X_1, \ X_2, \ X_3)$$

in which X_1 is the down-range distance, X_2 is the height above the 1-3 plane, X_3 is the lateral distance, positive to the right when looking down range, and $\dot{\vec{X}} = \vec{U}$. Since the trajectory (in most cases) lies nearly in the 1-2 plane, X_3 is usually much smaller in magnitude than X_1 and X_2.

Similarly, we can write

$$\begin{aligned}
\vec{U} &= (U_1, \ U_2, \ U_3) \\
\vec{V} &= (V_1, \ V_2, \ V_3) \\
\vec{W} &= (W_1, \ W_2, \ W_3) \\
\vec{G} &= (G_1, \ G_2, \ G_3).
\end{aligned}$$

in which U_3 is usually much smaller in magnitude than U_1 and U_2. The initial velocity is given by

$$\vec{U}_o = |\vec{U}_o|(\cos E, \ \sin E, \ 0)$$

in which E is the gun elevation. The details of the launching point are covered in the report of Cooper and Bradley (1991).

Eq.(1) can be written in component form as

$$\dot{U}_1 = (D-A)V_1 + \frac{1}{1+h_a}\left[G_1 + \frac{h_L(G_2 V_3 - G_3 V_2)}{(1-h_M)V}\right] \tag{5}$$

$$\dot{U}_2 = (D-A)V_2 + \frac{1}{1+h_a}\left[G_2 + \frac{h_L(G_3 V_1 - G_1 V_3)}{1-h_M V}\right] \tag{6}$$

$$\dot{U}_3 = (D-A)V_3 + \frac{1}{1+h_a}\left[G_3 + \frac{h_L(G_1 V_2 - G_2 V_1)}{(1-h_M)V}\right]. \tag{7}$$

The value of $\vec{G}$ is the sum of the gravity and Coriolis accelerations:

$$\vec{G} = \vec{g} + \vec{C} \tag{8}$$

4

The solution scheme used for this study has one important modification. Formerly, the influence equations did not include the yaw of repose; for this study, the influence equations must include the yaw of repose.

3. ATMOSPHERIC DATA AND MODELING OF ATMOSPHERE FOR FITTING DATA

In this study, we use the atmospheric conditions that are obtained from a weather balloon. We hope that the atmospheric model, which is assumed to fit the data, is close enough to reality to allow good approximations of the measured conditions. More complex models take more computer time and sometimes do not yield as good approximations as the more simple model. The latitude L for the simulated test was $39.15°$ North, the azimuth AZ was $21.915°$, and g_o was $9.80665\,m/s^2$.

3.1 Atmosphere Used in the Generating Equations To generate test trajectories, we used the balloon data taken at Aberdeen Proving Ground (APG) on 30 March 1992 in the early afternoon. The data consisted of pressure, temperature, and the wind velocity in the horizontal plane taken at very small intervals of altitude initially and increasing to intervals of 400 meters above altitudes of 3000 meters. The wind velocity data are shown in a later section when comparisons are be made with fitted values.

The temperature as a function of altitude is given in Figure 1.
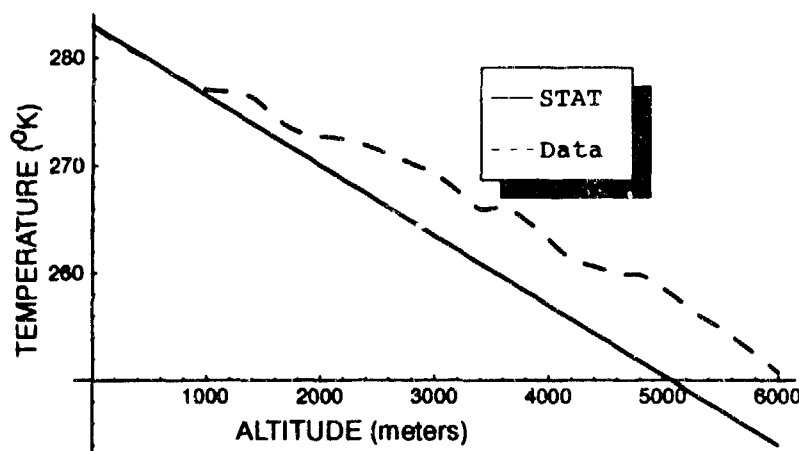
Figure 1. Temperature Data Compared to Standard Atmosphere Curve Temperature

Shown for comparison is the temperature curve obtained from the *U.S. Standard Atmosphere,*

1966, hereafter called STAT. The STAT curve can be considered as an approximation of the average temperature in temperate climates in North America. Although the recorded temperature varies no more than a few degrees from the STAT values, the temperature profile can vary markedly according to location and time of the year. The winter temperature profile in Siberia would have an initial positive gradient with respect to altitude, whereas the average temperature profile at the latitudes in North America would have a temperature gradient of opposite sign.

The density data are shown in Figure 2, together with the density curve obtained from STAT profile for locations at latitudes in North America. The density data, although not measured directly, are calculated from the pressure and temperature data. The trial input trajectory was obtained by numerically integrating the MPM equations with the interpolated balloon data.
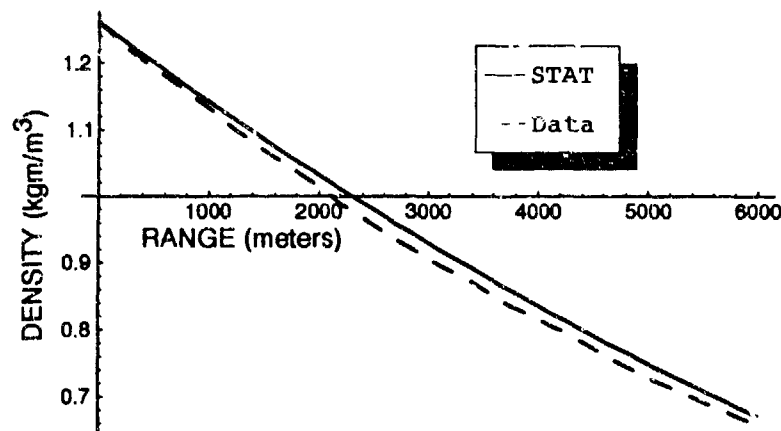
Figure 2. Density Data Compared to Standard Atmosphere Curve Density

3.2   Atmosphere Model used with the Fitting Equations Cooper and Bradley (1991) fitted the horizontal components of the wind, as well as the density and temperature, in one segment with a linear curve. Not surprisingly, they obtained a good fit with the imposed linear wind velocities. For the present work, we fitted the quantities within a few time segments. Initially, we fitted a linear relationship for the wind velocities within each segment, but we found that parameters, C_j, of constant value throughout each time segment gave a sufficient fit with the advantage of using fewer fitting parameters. Essentially, for each segment along the trajectory, we have for the wind velocity,

$$W_1 = C_1 \tag{9}$$

$$W_2 = 0 \tag{10}$$

$$W_3 = C_2 \tag{11}$$

The rate of change of temperature with respect to altitude, H, is the third parameter fitted for most of this study, $C_3 = T'$. Here, T' is the lapse rate with opposite sign. Cooper and Bradley (1991) assumed a temperature dependence on the height of the form

$$T = T_B + T'(H - H_B) \qquad (deg \ K) \tag{12}$$

in which the gradient of the temperature, T', is the negative of the lapse rate. The value H is called the geopotential altitude and takes the rotation of the earth into consideration. The geopotential altitude is discussed in more detail by Cooper and Bradley (1991). The value H_B is the value of H at the bottom of a geopotential zone for the STAT. For those zones in which T' is not zero, the STAT air density is given by

$$\frac{\rho}{\rho_B} = \left(\frac{T}{T_B}\right)^{-1-Q/T'} \tag{13}$$

in which the subscript B denotes the value of a variable at $H = H_B$, and

$$Q = \frac{g_0 \mathcal{M}}{\mathcal{R}}$$

in which $\mathcal{R}$ is the universal gas constant and $\mathcal{M}$ is the molecular weight. The value of Q for dry air is

$$Q_d = 34.16319474 \quad (deg \ K/km)$$

For moist air, the value of Q can be approximated as

$$Q_m = \frac{Q_d}{1 + 0.61\beta}$$

The value β is the mass fraction of water vapor in the air. This value of β can be as high as 0.04. For this study, we assumed that $\beta = 0$. A different expression is used in which the lapse rate is zero, but for this study, the lapse rate is never zero.

For the current study, the lapse rate and wind velocities were fitted on altitude zones assuming that the density varied as Equation (13). The subscript values at the bottom of the corresponding altitude zones are denoted as A. As before, the first time interval values of T_A, H_A, and ρ_A (obtained by measurements at the launch site) are required inputs to the fitting process; thereafter, the closing values for the k-th interval become the starting values for the (k+1)-th interval.

4. CHARACTERISTICS OF THE TEST PROJECTILE

We used an M107 projectile to generate trajectories. The M107 projectile has been well studied and all the coefficients needed for use with the MPM equations are well known (MacAllister and Krial 1975). This projectile has the following physical properties:

$$\ell \ (diameter) \ = \ 155 \ mm$$
$$m \ (mass) \ = \ 43.09 \ kg$$
$$I_x \ = \ 0.1461 \ kg - m^2$$

Of the six aerodynamic coefficients involved in our equations of motion,

$$C_{Do}, \ C_{D2}, \ C_{L_\alpha}, \ C_{M_\alpha}, \ C_{N_{p\alpha}}, \ \text{and} \ C_{\ell_p},$$

four are functions of Mach number, as indicated in Table 1.

Table 1. Mach Dependency of the Test Case Aerodynamic Coefficients

Mach	C_{Do}	Mach	C_{L_α}	Mach	C_{M_α}	Mach	C_{ℓ_p}
0.750	0.1150	0.6	1 650	0.69	3.30	0.50	-0.0145
0.825	0.1200	1.9	1.834	0.74	3.33	1.00	-0.0123
0.875	0.1310	1.5	2.294	0.79	3.40	2.00	-0.0096
0.910	0.1545	3.0	2.527	0.85	3.75	3.00	-0.0079
1.025	0.3800			0.93	4.67		
1.050	0.4170			0.97	3.90		
1.100	0.3900			0.99	3.70		
1.250	0.3660			1.04	3.56		
1.600	0.3250			1.10	3.51		
2.000	0.2900			3.00	2.61		
3.000	0.2200						

This table shows pairs of values: Mach number and corresponding coefficient value. Our code performs straight-line interpolation for Mach values between two entries. The remaining two aerodynamic coefficients are assumed to be constant:

$$C_{D2} \ = \ 4.0$$
$$C_{N_{p\alpha}} \ = \ -.75$$

The muzzle velocity for the M107 projectile was 397.4 m/s. The initial spin was taken to be 1148 rad/s.

Exactly the same aerodynamic behavior was assumed in the fitting equations as was used in the equations for generating the trajectories. For this caliber shell, it is generally conceded that the drag coefficient is known to within a half percent. The significance of such an error will be explored in the simulations.

5.   PRELIMINARY ANALYSIS

The least squares method needs a unique set of parameters that will yield a global minimum. A first consideration is whether the parameter solution of the flight dynamic equations are unique. The value A in Equation (1) contains C_D and the parameter ρ as cofactors. The value of C_D depends on the temperature, which has a parameter to be determined. Therefore, the value A would seem to contain two factors that can vary and still leave the value of A unchanged. However, the value of T depends upon ρ through a constraining equation, and uniqueness should be restored. Also, the parameter C_D is weakly influenced by other variables since C_D depends upon the yaw of repose.

Another consideration is the sensitivity of the parameter values to errors in measurement. Consider the shell at apogee, where the shell velocity is aligned approximately along the X_1 direction. With these conditions and using the notation of Cooper and Bradley (1991), the MPM model yields much simplified expressions which are, in component form,

$$\dot{U}_1 = -AV_1 \tag{14}$$

$$\dot{U}_2 = \frac{G_2}{1 + h_a} \tag{15}$$

$$\dot{U}_3 = \frac{h_L G_2}{(1 + h_a)(1 - h_M)}. \tag{16}$$

Assuming flat fire, we transform from the time to the X_1 coordinate and focus our attention on Equation (14)

$$U_1' = -a(1 - W_1/U_1)(U_1 - W_1) \tag{17}$$

in which $a = A/(V/\ell)$. This equation can be further simplified with the assumptions of no winds. Transforming to nondimensionalized variables, $x = (X_1 - X_{1b})/\ell$ and $\bar{t} = U_{1b}(t - t_b)/\ell$, and integrating we obtain

$$x = \frac{\ln{(1 + a\bar{t})}}{a}. \tag{18}$$

9

U_{1b} is the value of the first component of the velocity at the beginning of the trajectory segment and t_b is the value of t at the beginning of the segment. For a small enough segment the above equation can be approximated by expanding in series to two terms,

$$x = \bar{t}(1 - a\bar{t}/2). \tag{19}$$

The question of the accuracy with which one can determine ρ can be examined by using Eq. (19). Many of the measurement techniques smooth the output positions and velocities so that one is not aware of the fundamental error spread of the basic data. Given that this error in starting and ending positions is Δx, we want to find the required distance, x, of the segment to make the relative density error smaller than a certain value, or more generally, the relative error in a. We differentiate the logarithm of Equation (19) to obtain

$$\frac{\Delta x}{x} = -\frac{\bar{t}^2 \, \Delta a}{2(\bar{t} - a\bar{t}^2/2)}. \tag{20}$$

The quantity in parantheses in the denominator on the right-hand side is approximately equal to x for sufficiently small $\bar{t}$. Perform the substitution and rearrange to obtain

$$x = \sqrt{|2\,\Delta x / a\,(\Delta a/a)|}. \tag{21}$$

Here, $(\Delta a)/a$ is the relative uncertainty in a that can be tolerated, and Δx is the uncertainty in x attributable to measuring system limitations.

The value of C_{D_o} is generally not known to be better than a half percent. This uncertainty is partly attributable to limitations of measuring equipment for the aerodynamics range and partly to round-to-round differences. Thus, the relative error of a cannot be less than a half percent. The value of x is a minimum value that will give $\Delta a/a$ the allowed precision. A representative value of a for the M107 projectile at ground level for the numerical experiment is $a_0 = 1.3 \cdot 10^{-5}$. The drag coefficient decreases as the shell ascends and decelerates. Likewise, the density decreases with altitude and the value of a decreases accordingly. The resulting envelope for the minimum uncertainty for the trajectory length in which a relative uncertainty of a is $\leq 1\%$ is shown in Figure 3. For the test trajectory at or near apogee, the value of a is only about a fifth of its value on the ground. In practice, the relative uncertainty in a will be on the order of the relative uncertainty in the quantity ρ, which is to be found.

From Figure 3, it can be seen that the segment lengths for the M107 ($\ell = 0.155\,m$) must be $x \geq 13,000$ ($13,000 \times 0.155\,m \approx 2000\,m$) if the uncertainty in the segment length is 1 meter ($\Delta x = 1\,m/0.155\,m$) near apogee where the density times the drag coefficient is only a fifth of what it is at launch ($a/a_o = 0.2$). The length of such a segment is large, and the shell may change altitude more than is desired. To achieve this much precision in measuring the position may be a daunting task. Equation (21) is, of course, relevant to the spacing of the

data stations in a firing range. For a 7.62-mm projectile with the flight characteristics of an M107 shell, the segment length would need to be approximately 1 meter to obtain a relative accuracy of 0.005 in the drag coefficient. The range stations in the ARL Aerodynamic Range are placed at least a meter apart. Moreover, there are many stations along a considerable length. These features allow the Aerodynamics Range to achieve the desired accuracy.
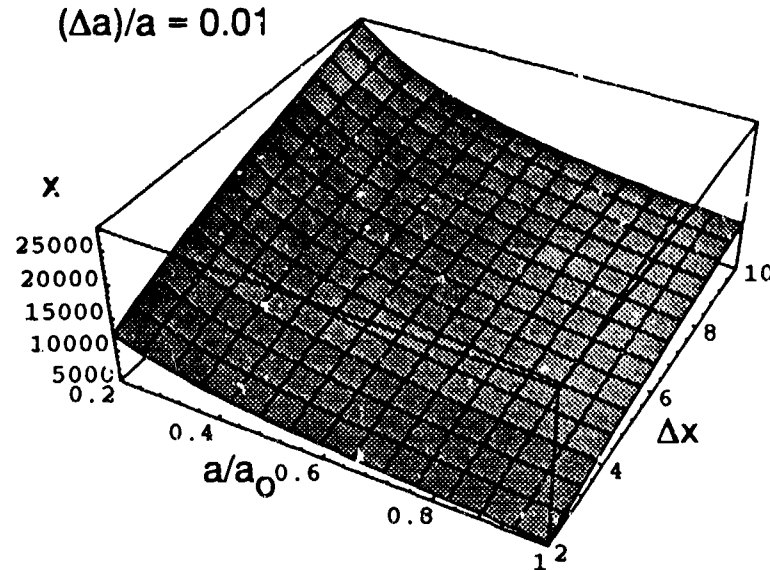

Figure 3. Required Segment length (cal) to Achieve Given Accuracy in a as a Function of a and Uncertainty in Segment Length

The relative effects of the density versus the effects of the change in wind velocity can be examined with the aid of Equation (17). The use of this equation is quite relevant since the shell spends a large amount of time near apogee, and thus, the trajectory should be significantly affected by changes in atmospheric values near apogee. The variation of its natural log, when set to zero, is equivalent to, for instance, changing the density and wind velocities so that the acceleration of the projectile remains unchanged. When the down-range wind velocity and a are free to vary, the resultant trajectory will not vary from its original path if

$$\frac{\delta a}{a} = \frac{2\,\delta W_1}{U_1 - W_1}. \tag{22}$$

11

6. TEST RESULTS AND DISCUSSION

The trajectory was fitted along its intervals in a sequential fashion, starting at ground level and using the ending values of the intervals as an initial estimate for the next interval. Although the initial estimates also serve as the unchanging initial values for the density and temperature, for the wind velocity values, initial estimates are adjusted to obtain the best fit along with the fitted lapse rate. The wind velocity data, together with the curves found by the least squares fitting method, are shown in Figure 4.
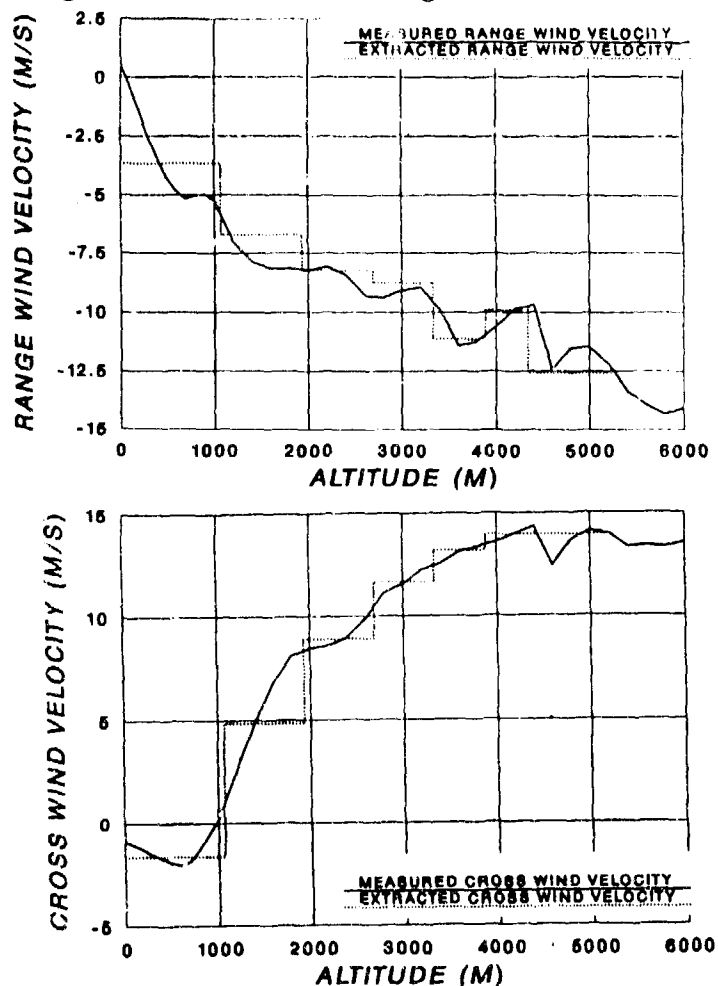
Figure 4. Comparison of Range Wind Velocity (W_1) and Cross Wind Velocity (W_3) Data with Respective Fitted Values

The profiles are roughly reminiscent of boundary layer behavior, as one would expect. The wind velocities are low near the ground and, at first, generally increase with altitude. The agreement of the least squares fitting results with the data is good, especially considering that a constant value of wind velocity is assumed for each segment. These fitted values, although not fitted on the same intervals as are used for firing tables, are in a format similar to the standard MET message.

12

The density and temperature data are compared with the fitted data in Figure 5. The fitted density agrees best with the data near the ground and agrees less well near apogee since the calculation is started at the lowest lying segment and proceeds stepwise upward with the ending density value for the lower segment becoming the initial value for the higher segment. Errors made near the beginning can propagate and amplify over the segments at higher altitudes. In fact, the fitted density values appear to diverge from the data at the higher altitudes.
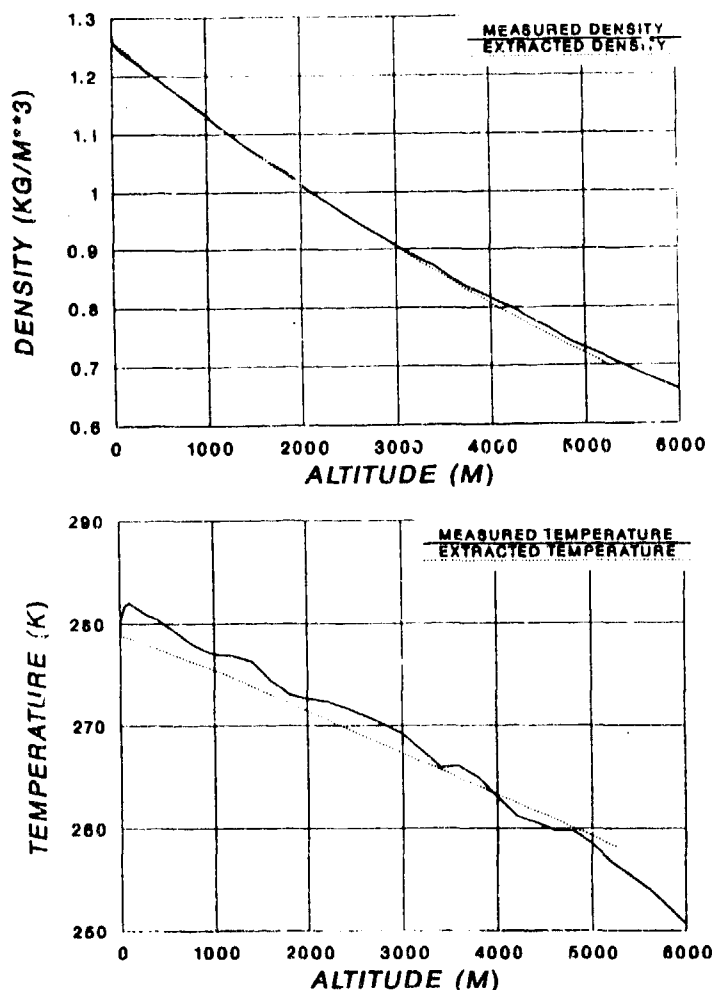

Figure 5. Comparison of Density and Temperature Data with Fitted Values

The temperature values obtained throughout the fitting calculations are erroneous by a few degrees over most of the trajectory. The initial value for the temperature is taken on the ground. A temperature inversion, which often occurs in the winter, results in a rapid initial increase in error with height. An error in the temperature chiefly affects the value of the zero-yaw drag coefficient, C_{Do}, which depends weakly on the temperature for supersonic velocities, more strongly for transonic velocities, and not at all for the low to moderate subsonic velocities. A major part of the trajectory occurs at subsonic velocities. Nevertheless, the first part of the trajectory occurs at supersonic and transonic velocities. The error in the temperature values also affects the density values since the change in the density depends upon the change in the temperature.

7. ERRORS INDUCED BY MEASUREMENT ERRORS AND MODEL ASSUMPTIONS

With the assumptions of a constant wind velocity and a linear temperature variation within a segment, it is obvious that the fitted trajectory could only approximate the trajectory that was generated by using real atmospheric data. Figure 6 shows the positional errors in meters as a function of altitude. The positional error is simply the difference between the fitted value of the position and the generated value of the position. Here, a large error occurs at a lower altitude with no spiking thereafter except for the maximum errors at the segments that occur at their end points. This large error at the lower altitudes is probably associated with the projectile being in the supersonic and transonic regions. If an error is made in obtaining the temperature, the drag coefficient can be significantly affected while in the subsonic region where the drag coefficient is only weakly dependent on the Mach number. Errors near the beginning of the segment must be small; otherwise, errors made near the beginning of the trajectory over the segment would tend to propagate and increase over the segment.
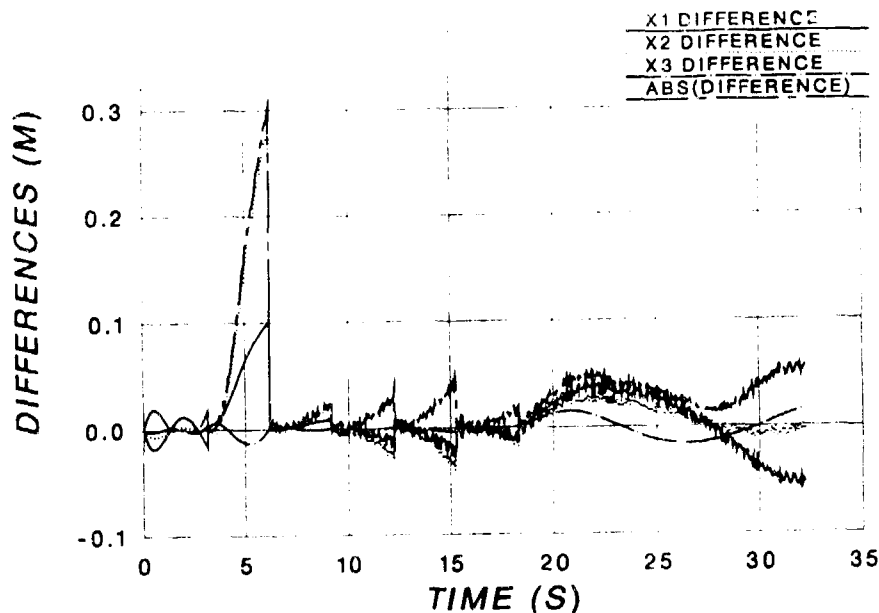

Figure 6. Error in Fitted Position (meters) *vs.* Altitude.

As discussed earlier, there will be an uncertainty in the measurements of the end positions of a trajectory segment which will induce errors in obtaining the fitted values of the meteorological quantities. Uncertainty in measured positions along the trajectory segment should also induce errors in fitting the atmospheric values. These uncertainties are explored here by superimposing Gaussian noise with a standard deviation (SD) of 0.2 meter distance upon the trajectory position points. To obtain a more detailed view of the error, we turn

14

to numerical analysis. It is shown that the number of significant digits, J, in the density is given as

$$J = \log_{10}\left(\frac{0.5}{E_\rho}\right), \tag{23}$$

in which E_ρ is the relative error of the density. For instance, Equation (23) shows that if the relative error, E_ρ, has the value 0.05, the number of significant digits, J, is equal to one. As the relative error becomes smaller, the number of significant digits becomes larger. The number of significant digits of resultant relative error in density for the fitted values, for both noise and no noise, are shown in Figure 7. Convergence to a solution took much longer with the noise present. Not surprisingly, the fitted solution with no noise results in a better fit. It also appears that the errors obtained at lower altitudes may propagate to degrade the solution at higher altitudes. As discussed earlier, however, longer segments are needed to obtain accurate fits as the altitude increases. Fitting the trajectory with Gaussian noise with a SD distance of 1 meter resulted in poor agreement and is not shown here.
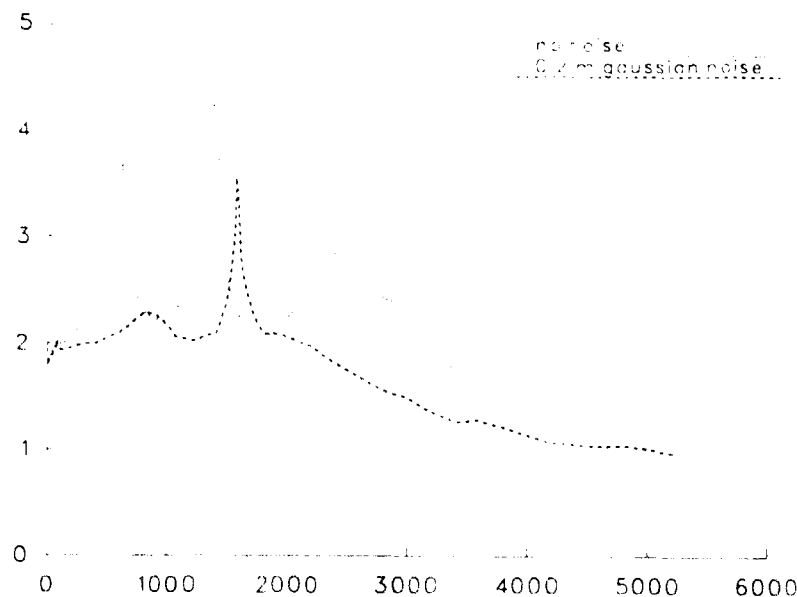

Figure 7. Significant Digits of Relative Error in Density *vs.* Altitude.

Another source of uncertainty is the actual value of the drag coefficient, C_D. The value of the drag coefficient is commonly known to within only a half percent, as discussed earlier. The drag coefficient value may also vary even more from lot to lot. To test this source of uncertainty, the fitted value of the drag coefficient was assumed to be a half percent lower than the drag coefficient values used in generating the trajectory. The resulting fitted density was approximately a half percent higher than had been obtained previously. This result would be expected.

The approach of using normally stale density and temperature data to obtain equivalent

winds can also be briefly scrutinized and perhaps be put in perspective. Table 2 shows the SD of atmospheric quantities measured at selected times after an initial measurement. This

Table 2. Errors Incurred in Stale Met Messages – M483A1 Shell

Staleness Hours	σ_{wind} m/s	σ_{den} %	σ_{temp} %	σ_{ran} m	σ_{defl} m
0	0.4	0.15	0.25	14	8
1	2.1	0.40	0.30	66	40
2	2.5	0.69	0.57	84	49
4	3.7	0.97	0.79	122	71
∞	5.7	6.60	3.0	341	109

table was generated from a report by Matts and F is (1990). Shown also are the associated error magnitudes for range and deflection for the M483A1 shell fired from the M109A2 howitzer. The target range is 15,000 meters with the zone 8 charge.

This table shows nonzero values with fresh atmospheric data. These values are equal to the instrument-measuring errors. The errors incurred by not taking into account atmospheric conditions are large, as shown in the last row of Table 2. If we assume that the density and temperature could be approximated by a standard atmospheric model, then using a shell's measured trajectory to find the wind velocity by a fitting process would yield huge fictitious wind velocities. If we use density and temperature data that are a few hours old, however, we can use Equation (22) to find the projectile velocity necessary to leave the net force of the projectile unchanged. For a staleness time of 4 hours, the value of the shell velocity to satisfy Equation (22) is approximately 740 m/s. This velocity will be higher than the the actual velocity of the shell at apogee and shows that the error in the density contributes more to the range error than is contributed by the error in wind velocity. Pedersen (1994) is exploring the possibility of using more recent ground observations to make an assertion about how atmospheric conditions have changed at higher altitudes. Such knowledge would avoid the large errors in range and deflection that might occur if the artillery piece were fired on a different azimuth.

Measurements over the entire trajectory have not been available, and particularly, measurements have not been available over the trajectory portion near the ground. Although the current measurements appear as a smooth curve, the smoothness is an artifice of the data reduction technique. For both GPS and radar systems, the actual uncertainty can be meters. According to the analysis made earlier, the uncertainty in the atmospheric quantities would

be unacceptably large for the segment lengths used here.

Other approaches are obvious to obtain sufficiently accurate atmospheric values but will only be discussed here. Accuracy should be improved by using the whole trajectory length although the calculational effort would be much increased. The atmospheric values would have to match at any given altitude while both ascending and descending, thus increasing the number of constraints on the solution. The density should be fitted as a continuous function while the fitted wind velocities would be assumed as constant values over segments of the trajectory. This method would be slow to converge to a solution, and the accuracy of the fitted values might still be affected by the drag coefficient and the velocity appearing in the flight dynamic equations as complementary factors. As for the current approach, a bad guess for the initial estimates may result in a converged solution that is far from the absolute minimum.

8. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

By using a least squares fitting procedure, we obtained atmospheric conditions knowing only the projectile's trajectory and its aerodynamic coefficients. The projectile trajectory was generated using the MPM model. The atmospheric conditions were supplied by a weather balloon. A least squares fit of several trajectory segments that comprised altitude zones was made. The fitting procedure started from the ground and used the fitted density and temperature conditions at the end point of the trajectory segment as the initial fitted conditions for the next segment. The agreement of the fitted meteorological values with the balloon data was good, but the agreement decreased with the increasing amounts of noise added to the trajectory positions. A superimposed noise with a SD of 1 meter resulted in an unsatisfactory fit. These results agree with an error analysis. To obtain atmospheric conditions with the needed precision, the accuracy study also shows that the starting and ending positions of segment lengths must be known with more precision than current measuring techniques are capable of providing. A different approach is needed, such as using the data over the whole trajectory instead of fitting to the data from a relatively small trajectory segment.

The calculational effort would be dramatically reduced if the temperature were known, and hence the temperature dependent flight coefficients, such as the drag coefficient, would be immediately known. A thermometric device on board a flight projectile could radio the temperature data to a ground station. Although the hardware complexity would be somewhat increased, this approach might yield a practical robust solution with rapid convergence. Calculations could be accelerated even more if the density could be determined from the assumption of a hydrostatic atmosphere, knowledge of the temperature, and the equation of

state. To obtain the equation of state, however, the moisture content of the air must be known. Generally, the moisture content of the air is appreciable near the ground. Perhaps, one could use the current moisture content on the ground together with older detailed data to supply data with sufficient accuracy.

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LIST OF SYMBOLS

a $A/(V/\ell)$

A $\left(\dfrac{\rho S\ell}{2m}C_D\right)\left(\dfrac{V}{\ell}\right),$ $[1/s]$

AZ azimuth of the 1-axis, measured clockwise from North

B $k_a^{-2}\left(\dfrac{\rho S\ell}{2m}\right)\left(\dfrac{V}{\ell}\right)^2,$ $[1/s^2]$

$\vec{C}$ Coriolis acceleration, $-2\vec{\omega}\times\vec{U}$

C_D drag coefficient: $|drag\ force| = (\rho V^2 S/2)C_D$

$C_{Do},\ C_{D2}$ zero-yaw and yaw-drag coefficients:
$$C_D = C_{Do} + C_{D2}|\vec{\alpha}_e|^2$$

C_{ℓ_p} roll damping moment coefficient:
$$|roll\ damping\ moment| = \pm(\rho V^2 S\ell/2)(\dot{\phi}\ell/V)C_{\ell_p}$$

C_{L_α} lift force coefficient:
$$|lift\ force| = \pm(\rho V^2 S/2)|\vec{\alpha}_e|C_{L_\alpha}$$

C_{M_α} static moment coefficient:
$$|static\ moment| = \pm(\rho V^2 S\ell/2)|\vec{\alpha}_e|C_{M_\alpha}$$

$C_{N_{p\alpha}}$ Magnus force coefficient:
$$|Magnus\ force| = \pm(\rho V^2 S/2)(\dot{\phi}\ell/V)|\vec{\alpha}_e|C_{N_{p\alpha}}$$

$C_1, ...C_3$ fitting parameters defining the wind, Eqs.(9) and (11)

D $\dfrac{h_a(\vec{G}\bullet\vec{V})}{(1+h_a)V^2},$ $[1/s]$

E gun elevation

E_ρ relative error in the fitted air density

g $|\vec{g}|$

$\vec{g}$ gravity acceleration

g_o $|\vec{g}|$ at sea level; the STAT value at 45 deg. N. latitude is 9.80665 m/s

$\vec{G}$ $\vec{g} + \vec{C}$, gravity plus Coriolis acceleration

$\vec{G}_A$ $\frac{1}{1+h_a}\left[\vec{G} + \frac{h_L(\vec{G}\times\vec{V})}{(1-h_M)V}\right]$, $[m/s^2]$

G_1, G_2, G_3 flat-earth system components of $\vec{G}$

h_a $\frac{h_L^2}{1-h_M} - h_M$

h_L $k_a^2\left(\frac{C_{L_\alpha}}{C_{M_\alpha}}\right)\left(\frac{\dot{\phi}\ell}{V}\right)$

h_M $k_a^2\left(\frac{C_{N_{p\alpha}}}{C_{M_\alpha}}\right)\left(\frac{\dot{\phi}\ell}{V}\right)^2$

H geopotential altitude, Eq.(12)

H_A value of H at the starting time of a fitting interval

H_B value of H at the bottom of a STAT altitude zone (Table 1)

I_x axial moment of inertia

J number of significant digits in the fitted air density

k_a^2 $I_x/m\ell^2$

ℓ reference length

L latitude at the launch point (for Southern Hemisphere firings, replace L by $-L$)

m	projectile mass
$\mathcal{M}$	molecular weight
Q	constant in the STAT air density formula, (13)
R	effective radius of the earth (6,356,766 m)
$\mathcal{R}$	universal gas constant
S	reference area, $\pi \ell^2 / 4$
STAT	1966 U.S. Standard Atmosphere, referenced
t	time
$\bar{t}$	nondimensionalized time, $U_{1b}(t - t_b)/\ell$
T	temperature, degrees Kelvin
T'	temperature gradient dT/dH; fitting parameter C_3
T_A	temperature at the starting time of a fitting interval
T_B	STAT temperature at altitude H_B
$\vec{U}$	projectile velocity with respect to the earth
U_1, U_2, U_3	flat-earth system components of $\vec{U}$
V	$|\vec{V}|$
$\vec{V}$	$\vec{U} - \vec{W}$, projectile velocity with respect to the air
V_s	speed of sound

$V_1,\ V_2,\ V_3$	flat-earth system components of $\vec{V}$
$\vec{W}$	wind velocity with respect to the earth
$W_1,\ W_2,\ W_3$	flat-earth system components of $\vec{W}$
$\vec{X}$	projectile position with respect to the earth
x	nondimensionalized position variable, $(X_1 - X_{1b})/\ell$
$X_1,\ X_2,\ X_3$	flat-earth system components of $\vec{X}$
	X_1 : down range
	X_2 : the height above sea level
	X_3 : lateral, positive to the right looking down-range

$\vec{\alpha}_e$	the yaw of repose
β	the mass fraction of water vapor in the air
ρ	air density
ρ_A	air density at the starting time of a fitting interval
ρ_B	STAT air density at H_B, see Table 1
$\dot{\phi}$	axial spin rate
$\vec{\omega}$	angular velocity of the earth
ω_E	$|\vec{\omega}|$
$\omega_1,\ \omega_2,\ \omega_3$	flat-earth components of $\vec{\omega}$:
	$\omega_E(\cos L\ \cos AZ,\ \sin L,\ -\cos L\ \sin AZ)$
$\dot{(\ )}$	$d(\ )/dt$

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